Written Addition and Subtraction Strategies

Progression of the methods taught and learned by children from Reception to Year 6 at Awbridge and Wellow Primary Schools

These strategies are an agreed hierarchical progression of written methods. They conform to the statutory national end of year expectations for Year R through to Year 6. Within a challenging mathematical environment, the learner will move on, often with smaller 'sub-steps' taken in between the main ones shown in this booklet.

Early steps...

These are the minimum end of year expectations for YR.

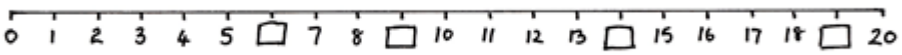
- **Count reliably to 20.**
- **Order numbers 1 – 20.**
- **Say 1 more/1 less to 20.**
- **Add & subtract two single digit numbers.**

Many children in our schools will meet and exceed these statements and will be working within the statements for Y1; much of their learning will be through practical activities;

For example:

- Talk about, recognise and recreate simple patterns
- Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices
- Say and use number names in order in familiar contexts
- Know that numbers identify how many objects are in a set
- Estimate how many objects they can see and check by counting
- Count aloud in ones, twos, fives or tens
- Count repeated groups of the same size
- Observe number relationships and patterns in the environment and use these to derive facts
- Begin to use the vocabulary involved in division, ie. group, set, share.

Written number tracks are used alongside practical activities so that children begin to make the links between concrete and abstract mathematical ideas



Next steps in KS1

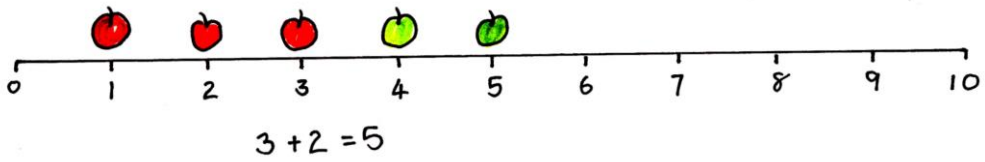
Minimum end of year expectations for Y1

- Count to & across 100, forwards & backwards from any number.
- Read & write numbers to 20 in digits & words.
- Read & write numbers to 100 in digits.
- Say 1 more/1 less to 100.
- Know bonds to 10 by heart.
- Use bonds & subtraction facts to 20.
- Add & subtract:
- 1 digit & 2 digit numbers to 20, including zero.
- Add any three 1-digit numbers with a total up to 20.

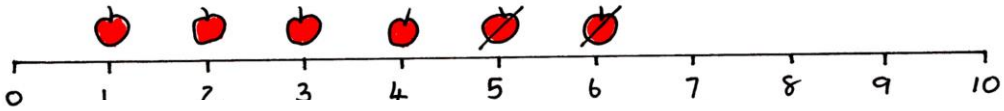
We make links with the practical activity of counting how many objects there are altogether (addition) or taking away objects from a group (subtraction), to jumps on a number line.

For example, 3 + 2 can be shown as:

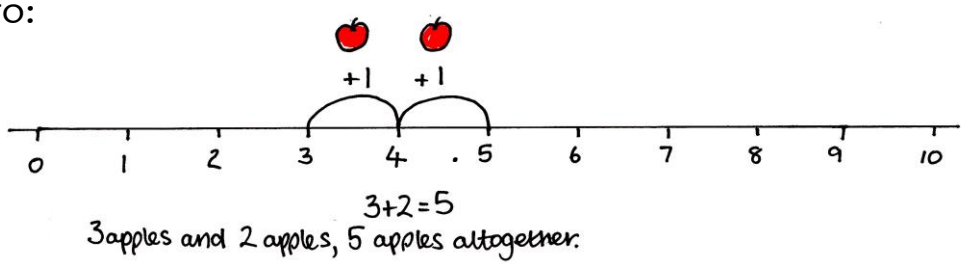
If I have 3 apples and 2 apples, I have 5 apples altogether

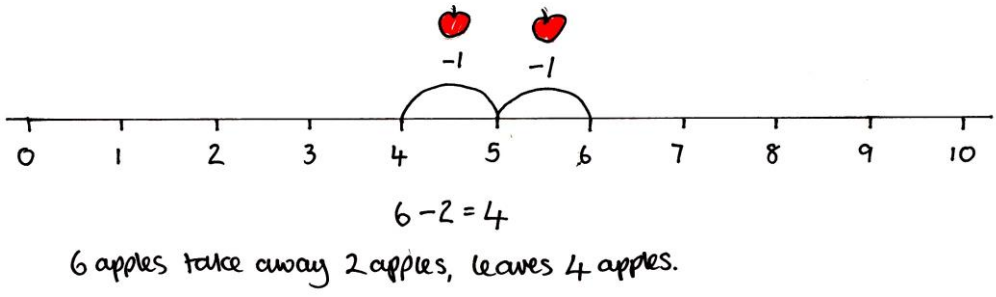


If I have 6 apples and I eat 2, I have 4 apples left.



Leading on to:





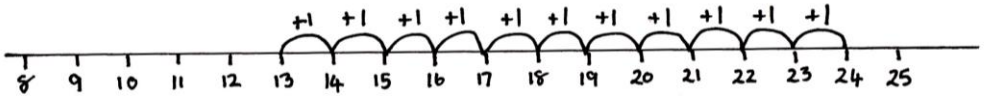
Minimum end of year expectations for Y2

- Compare & order numbers up to 100.
- Read & write all numbers to 100 in digits & words.
- Say 10 more/less than any number to 100.
- Recall & use +/- facts to 20.
- Derive & use related facts to 100.
- Recognise PV of any 2-digit number.
- Add & subtract:
- 2-digit numbers & ones
- 2-digit numbers & tens
- Two 2-digit numbers
- Three 1-digit numbers
- Recognise & use inverse (+/-).

A structured number line is modelled and used by the children to support counting and calculating, through meaningful contexts.
Children begin to use the structured number line independently as a tool to support mental calculation. Alongside this they use number sentences to record the calculation. We use the structured number line to support early place value knowledge i.e. addition of 2 'teen numbers' by partitioning one number and counting on in tens and ones.

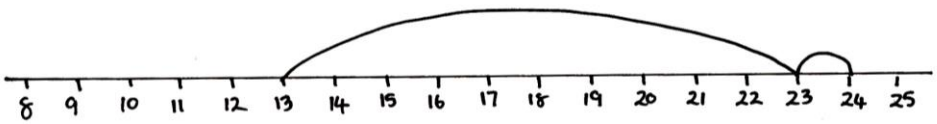
For example, 13 + 11 can be calculated as:

"If a farmer has 13 apples on one tree and 11 apples on another. How many apples does he have altogether?"



13 apples and 11 apples equals 24 apples.

13 + 11 = 24

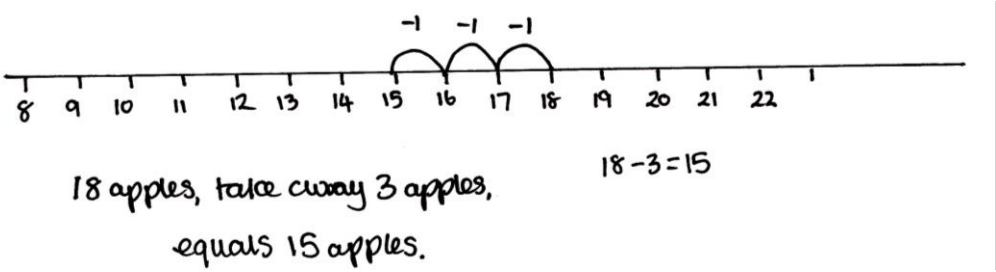


13 apples and 10 apples, and 1 apple, equals 24 apples.

13 + 11 = 13 + 10 + 1
13 + 10 = 23
23 + 1 = 24

Likewise, the number line can be used to support subtraction of single digit numbers from two digit numbers:

For example, 18 – 3 can be calculated as:
"If a farmer has 18 apples on a tree and he picks 3, how many apples will be left?"

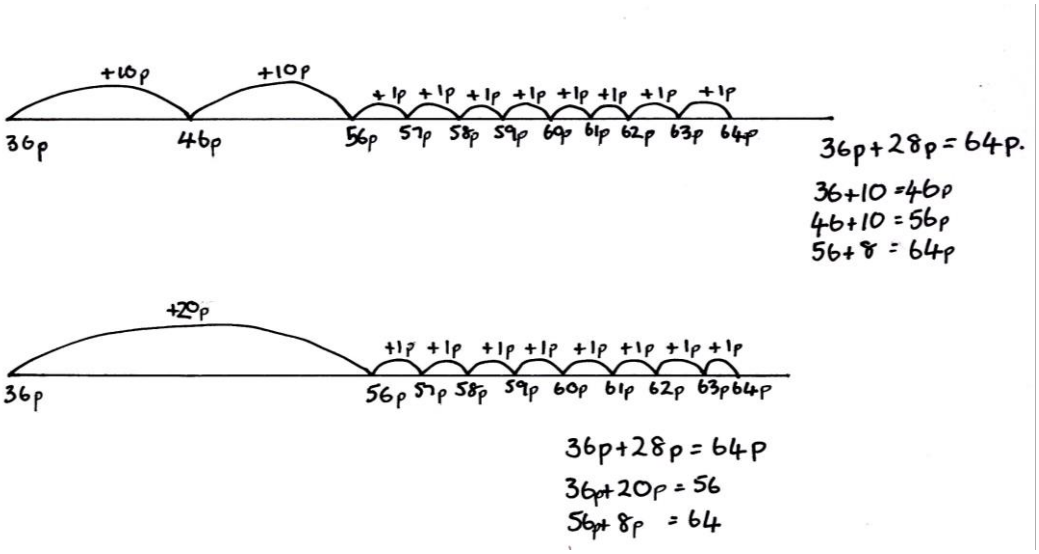


Continued use of a structured and then an unstructured number line is used to support counting and calculating and to develop children's understanding of partitioning to add larger 2 digit numbers.

In a structured number line, all of the numbers are pre-recorded; in the unstructured number line, children record their own jumps in calculation.

*For example: W*hen adding two - digit numbers by partitioning one number and counting on in tens and ones then multiples of ten and ones, it could be modelled as:

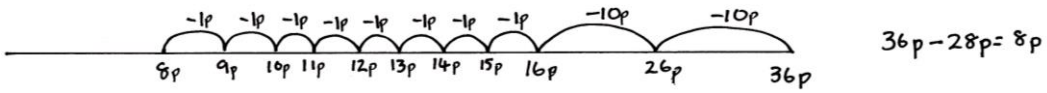
"I have 36p and my mum gives me 28p pocket money. How much money do I have altogether?"



Real life problems are an integral part of learning how to subtract at all stages in the learning process, and real materials such as money are used to help with the calculation and to make the experience meaningful.
For example, when subtracting two, 2 - digit numbers by partitioning the second number and counting back in tens and ones then multiples of ten and ones.

It could be modelled as:

"I have 36p, I spend 28p. How much do I have left?"



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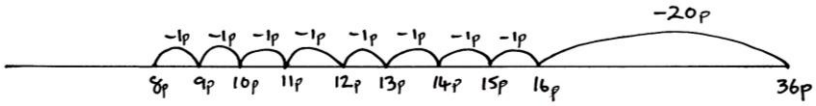


The children are shown how they can use the relationship between addition and subtraction to help with their calculations.
For example :

from 2 + 4 = 6 they can derive the inverse will be 6 - 4 = 2,

and from 25 + 13 = 38 recognise that 38 - 13 = 25.

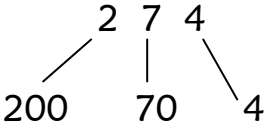
Moving on in Lower KS2

Minimum end of year expectations for Y3

- Compare & order numbers up to 1000
- Read & write all numbers to 1000 in digits & words.
- Find 10 or 100 more/less than a given number.
- Recognise PV of any 3-digit number.
- Add & subtract:
  - 3-digit numbers & ones
  - 3-digit numbers & tens
  - 3-digit numbers & hundreds
- Add & subtract:
  - Numbers with up to 3-digits using efficient written method (column).
- Use inverse operations to check.

Children continue to develop their understanding of place value and are able to partition the components of 3 digit numbers.

For example:

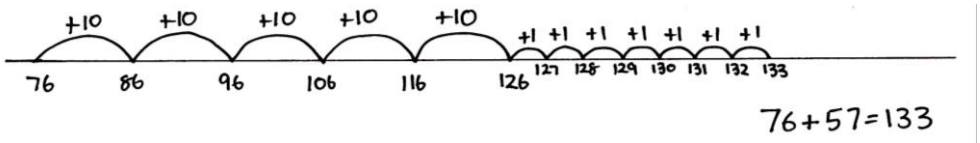


They will continue to use unstructured number lines as a tool to record their thinking during addition or subtraction calculations and to secure their understanding of subtraction as the inverse of addition.

Their recording will develop so they cross tens and hundreds boundaries accurately. They will partition numbers in different ways to record and make their calculations more efficient.

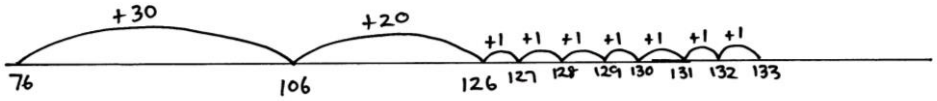
For example: When crossing the hundreds boundary, they could record it like this:

"There are 76 marbles in one jar and 57 marbles in another jar.
How many marbles are there altogether?"



76+57=133

Then…



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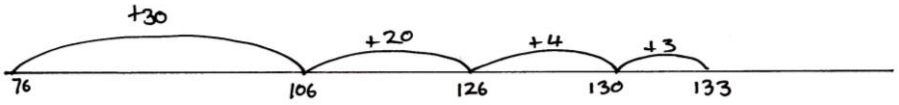
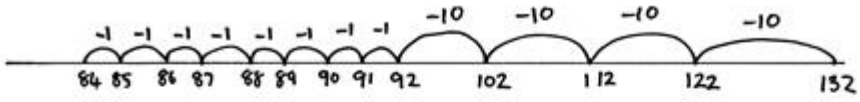


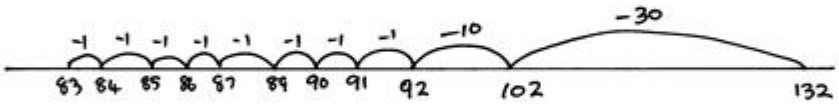
Likewise, for subtraction:

"There were 132 marbles in a jar. Sarah took 48 out of the jar.
How many marbles were left?"



132-48

Then…



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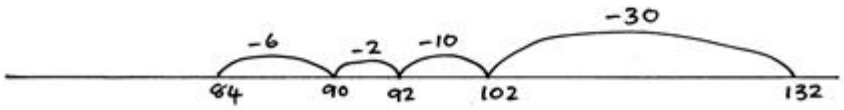


Leading on to adding two 2-digit numbers by partitioning without number line support:

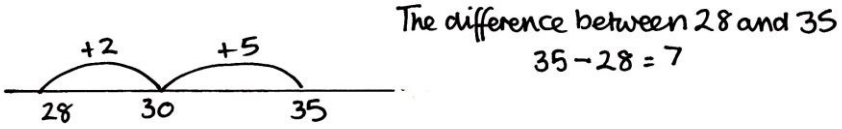
$$67 + 24 = (60 + 20) + (7 + 4) = 80 + 11 = 91$$

Subtraction as Difference…..

When children understand the concept of difference, through practical activity and can confidently subtract by counting backwards, they are ready to begin to use 'counting on' to find the difference **if the numbers are close together** e.g. difference between 28 and 35. Some more able children may be ready to use this strategy much earlier. Children are encouraged to look at the numbers in a calculation and decide for themselves whether it is better to count on, or to count back.

For example:

"Sam has 28 pencils and Sarah has 35 pencils. How many more pencils does Sam need to have the same number of pencils as Sarah?"



When adding or subtracting larger numbers, and when children are secure with place value, partitioning with 2-digit and 3-digit numbers becomes a more efficient way of calculating. We place the hundreds under the hundreds, the tens under the tens and the units under the units, in preparation for more formal column methods.

For example:

$$\begin{aligned} 76 + 47 &= 70 + 6 \\ &\quad + 40 + 7 \\ &\quad 110 + 13 = 123 \end{aligned}$$

Or

$$\begin{aligned} 258 + 164 &= 200 + 50 + 8 \\ &\quad + 100 + 60 + 4 \\ &\quad 300 + 110 + 12 = 422 \end{aligned}$$

Similarly, for subtraction of larger numbers:

$$\begin{aligned} 57 - 24 &= 50 + 7 \\ &\quad - 20 + 4 \\ &\quad 30 + 3 = 33 \end{aligned}$$

Or

$$\begin{aligned} 648 - 316 &= 600 + 40 + 8 \\ &\quad - 300 + 10 + 6 \\ &\quad 300 + 30 + 2 + 332 \end{aligned}$$

However, sometimes we need to 'exchange' to make a column sum work. As always, we model this concept with equipment such as counting materials, HTU boards, or money, alongside the algorithm:

Decomposition:

Using partitioning (separating the hundreds, tens and units) the children learn how to **exchange** tens for units, or hundreds for tens, etc. so they can successfully complete their subtraction algorithm:

H	T	U		H	T	U		H	T	U	
5	4	8	→	500	+ 40	+ 8	→	500	+ 30	+ 18	
- 1	2	9		- 100	+ 20	+ 9		- 100	+ 20	+ 9	
								400	+ 10	+ 9	= 419

We make it very clear that we are NOT 'borrowing' and 'paying back', but EXCHANGING for different denominations: for example, 1 ten for 10 units ; or 1 hundred for 10 tens.

Minimum end of year expectations for Y4

- Find 1000 more/less than a given number.
- Recognise PV of any 4-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Round decimals with 1decimal place (dp) to nearest whole number.
- Add & subtract:
  - Numbers with up to 4-digits using efficient written method
  - Numbers with up to 1dp.

We progress to a more compact form of recording our column sums, with larger (Th, H, T, U) and smaller (decimals to 1 place) numbers When the column total is a number with 2 digits, we 'carry' the ten, or hundred.
For example :

Th	H	T	U
	2	4	1 6
+ 1	.	6	0 4
4	0	2	0
1		1	

T	U	.	d
3	5	.	4
+ 2	9	.	1
6	4	.	5
1			

Children are taught to record compact column methods for subtraction too, showing their exchanges when necessary.
For example :

Th	H	T	U
		4	1
5	7	~~5~~	8
-2	7	0	9
3	0	4	9

T	U	.	d
	4		1
2	~~5~~	.	1
-1	3	.	5
1	1	.	6

Moving on in Upper KS2

Minimum end of year expectations for Year 5

- **Compare & order numbers with 3 decimal places (dp) .**
- **Recognise PV of any number up to 1000000.**
- **Round any number up to 1000000 to the nearest 10, 100, 1000, 10000 or 100000.**
- **Round decimals with 2dp to nearest whole number & 1dp.**
- **Add & subtract:**
  - **Numbers with more than 4-digits using efficient written method (column).**
  - **Numbers with up to 2dp.**

Children extend their understanding of place value to large numbers up to one million and small numbers with up to 2 decimal places. They need to be secure with this when using column methods to calculate addition or subtraction sums where the numbers have different numbers of digits.
For example:
We extend to larger numbers:
Adding the least significant (smallest) digits first, preparing for carrying:

8 + 3 = 11 50 + 70 = 120 300 + 0 = 300

	3	5 8
+		7 3
	1	1
1	2	0
	3	0 0
	4	3 1

leading to 'carrying' below the line:

$$\begin{array}{r} 358 \\ +\ 73 \\ \hline 431 \\ \scriptstyle 1\ 1 \end{array}$$

We extend to larger numbers and decimals:

$$\begin{array}{r} 2559 \\ +\ 1472 \\ \hline 4031 \\ \scriptstyle 1\ 1\ 1 \end{array} \qquad \begin{array}{r} 72.5 \\ +\ 59.6 \\ \hline 132.1 \\ \scriptstyle 1\ 1 \end{array}$$

Minimum end of year expectations for Year 6

There are no new algorithms for year 6; the children apply what they have learned in the past to multi- operation, multi-step problems.

Bibliography:

'The National Curriculum in England key stages 1 and 2 framework document' in September 2013.
Progression in the Use of the Number line – Hampshire Mathematics Advisory Team
The National Numeracy Framework Exemplars - DCSF

Sally Buckland 2014