



Written Multiplication and Division Strategies



Progression of the methods taught and learned by children from Reception to Year 6 at Awbridge and Wellow Primary Schools

These strategies are an agreed hierarchical progression of written methods. They conform to the statutory national end of year expectations for Year R through to Year 6. Within a challenging mathematical environment, the learner will move on, often with smaller 'sub-steps' taken in between the main ones shown in this booklet.

Early steps...

These are the minimum end of year expectations for YR.

- Count reliably to 20.
- Order numbers 1 – 20.
- Say 1 more/1 less to 20.
- Add & subtract two single digit numbers.

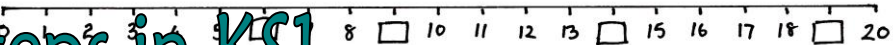
Many children in our schools will meet and exceed these statements and will be working within the statements for Y1; much of their learning will be through practical activities;

For example:

- Talk about, recognise and recreate simple patterns
- Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices
- Say and use number names in order in familiar contexts
- Know that numbers identify how many objects are in a set
- Estimate how many objects they can see and check by counting
- Count aloud in ones, twos, fives or tens
- Count repeated groups of the same size
- Observe number relationships and patterns in the environment and use these to derive facts
- Begin to use the vocabulary involved in division, ie. group, set, share.

Written number tracks are used alongside practical activities so that children begin to make the links between concrete and abstract mathematical ideas.

Next steps in KS1



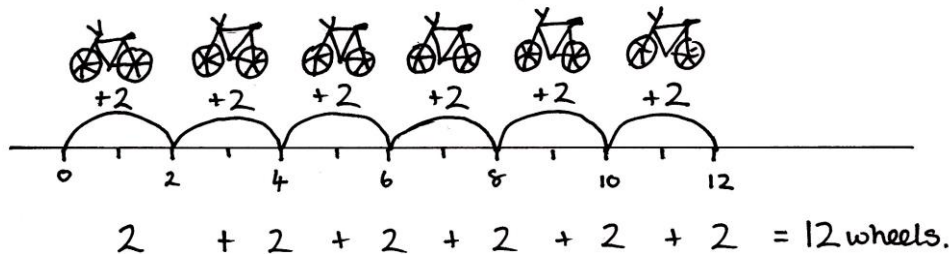
Minimum end of year expectations for Y1

- Count to & across 100, forwards & backwards from any number.
- Read & write numbers to 20 in digits & words.
- Read & write numbers to 100 in digits.
- Say 1 more/1 less to 100.
- Count in multiples of 1, 2, 5 & 10.
- Solve simple multiplication & division with apparatus & arrays.
- Recognise half and quarter of object, shape or quantity.

A structured number line is modelled and used by the children to support counting and calculating, through meaningful contexts.

For example:

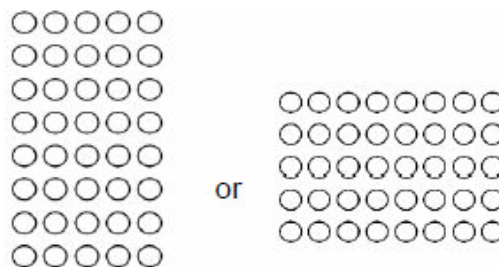
For '6 x 2', children picture:



Or they can picture multiplication as an array.

For example:

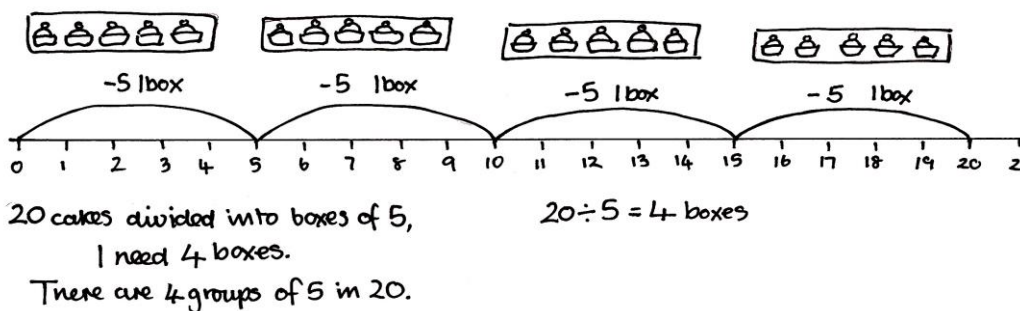
For '8 x 5', children picture:



These methods also support children's understanding of division as grouping or repeated subtraction. Explicit links are constantly made between the recording on the number line or array, and the practical activity.

For example :

$20 \div 5 =$ I have 20 cakes; I can fit 5 cakes in a box. How many boxes will I need?



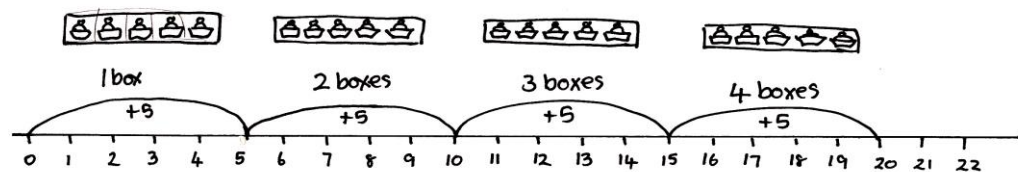
Minimum end of year expectations for Y2

- Compare & order numbers up to 100.

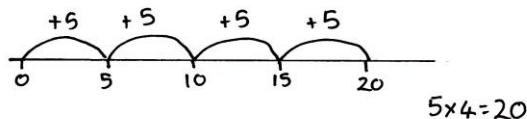
- Read & write all numbers to 100 in digits & words.
- Say 10 more/less than any number to 100.
- Count in multiples of 2, 3 & 5 & 10 from any number up to 100.
- Recall & use multiplication & division facts for 2, 5 & 10 tables.
- Recall & use +/- facts to 20.
- Derive & use related facts to 100.
- Recognise PV of any 2-digit number.
- Calculate & write multiplication & division calculations using multiplication tables.
- Recognise & use inverse (\times/\div).
- Recognise, find, name & write $\frac{1}{3}$; $\frac{1}{4}$; $\frac{2}{4}$; $\frac{3}{4}$.
- Show that multiplication of two numbers can be done in any order (commutative law) but division of one number by another cannot.

Continued use of a structured and then an unstructured number line is used to support counting and calculating and to develop children's understanding of multiplication as repeated addition.

For example:



$5 \times 4 = 20$
5 cakes in each box. 4 boxes equals 20 cakes altogether.

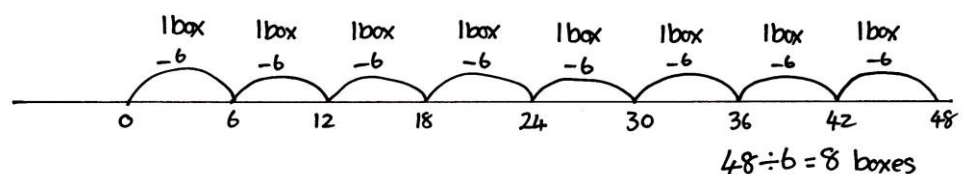


In a structured number line, all of the numbers are pre-recorded; in the unstructured number line, children record their own jumps in calculation.

The same methods are modelled and used to calculate division as grouping or repeated subtraction through making links to counting back in steps of equal size. (multiples of 2, 3, 4, 5, 6's and 10's). Children would begin with a structured number line then move on to making their own decisions.

For example:

$$48 \div 6 =$$

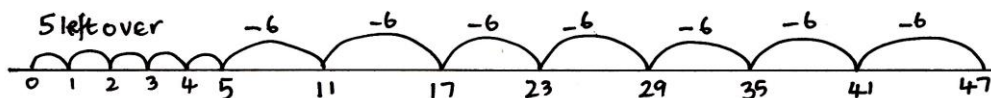


$48 \div 6 = 8$ boxes

Children begin to

understand that some division calculations have remainders and make a decision about rounding up or down according to the context:

I have 47 cakes. I can fit 6 cakes in a box. How many boxes will I need?



$47 \div 6 = 7$ boxes, with 5 cakes leftover.
You might need 8 boxes!

Real life problems are an integral part of learning how to divide at all stages in the learning process, and real materials such as money are used to help with the calculation and to make the experience meaningful.

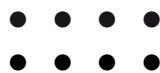
The children are shown how they can use the relationship between multiplication and division to help with their calculations.

For example :

from $2 \times 4 = 8$ they can derive the inverse will be $8 \div 4 = 2$,

and from $25 \times 3 = 75$ recognise that $75 \div 3 = 25$.

We begin to recognise from arranging arrays that multiplication can be done in any order (commutative law).



$4 \times 2 = 8$ $2 \times 4 = 8$

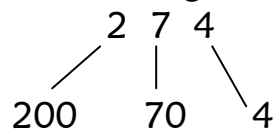
Moving on in Lower KS2

Minimum end of year expectations for Y3

- Compare & order numbers up to 1000.
- Read & write all numbers to 1000 in digits & words.
- Find 10 or 100 more/less than a given number.
- Count from 0 in multiples of 4, 8, 50 & 100.
- Recall & use multiplication & division facts for 3, 4, 8 tables.
- Recognise PV of any 3-digit number.
- Use inverse to check.
- Multiply: 2-digit by 1-digit numbers

Children continue to develop their understanding of place value and are able to partition the components of 3 digit numbers.

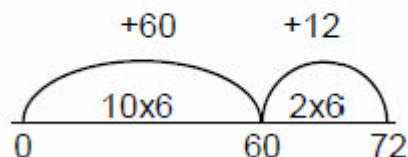
For example:



They will continue to use unstructured number lines as a tool to record their thinking during multiplication or division calculations and to secure their understanding of division as the inverse of multiplication. Their recording will develop so they cross tens and hundreds boundaries accurately. They will partition numbers in different ways to record and make their calculations more efficient, using their knowledge of multiplication tables.

For example:

$$12 \times 6 =$$



Or:

$$12 \times 6 = \quad 10 \times 6 = \underline{60} \quad + \quad 2 \times 6 = \underline{12}$$

$$\underline{60} + \underline{12} = 72$$

$$\text{So: } 12 \times 6 = 72$$

Once children are secure in their understanding of partitioning numbers, multiplication and division, they will be introduced to the grid method of multiplication:

$$12 \times 6 =$$

x	10	2
6	60	12

$$60 + 12 = 72$$

$$\text{So } 12 \times 6 = 72$$

Children will use halving and doubling to solve problems with numbers up to 100 and begin to multiply 2 digit by 1 digit numbers mentally.

Minimum end of year expectations for Y4

- Find 1000 more/less than a given number.
- Count in multiples of 6, 7, 9, 25 & 1000.
- Recall & use multiplication & division facts all tables to 12x12.
- Count up/down in hundredths.
- Multiply: 2-digit by 1-digit numbers 3-digit by 1-digit numbers
- Divide: 3-digit by 1-digit numbers

Children continue to use the grid method to partition and multiply 2 and 3 digit numbers by 1 digit numbers, using their knowledge of the multiplication tables.

$248 \times 5:$

<i>x</i>	<i>200</i>	<i>40</i>	<i>8</i>
<i>5</i>	<i>1,000</i>	<i>200</i>	<i>40</i>

$1,000+200+40=$

$1,240$

$248 \times 5 = 1,240$

As they secure their understanding, they will begin to record using a column method:

$$\begin{array}{r}
 248 \\
 \times \quad 5 \\
 \hline
 40 \quad (5 \times 8) \\
 200 \quad (5 \times 40) \\
 1000 \quad (5 \times 200) \\
 \hline
 1240
 \end{array}$$

$248 \times 5 = 1,240$

Children describe what they are doing by referring to the value of the digits so they say, '5 x 40' not '5 x 4' and '5 x 200' not '5 x 2'.

Column Method for Division: 'Chunking'

The method of 'chunking', or repeated subtraction of groups of the same number, is a direct progression of the number line work the children have now become confident with; we begin to record in a more formal way, using an expanded vertical method and showing the 'chunks' of the divisor that have been subtracted:

$$\begin{array}{r}
 27 \div 9 \quad \text{becomes} \\
 \begin{array}{r}
 27 \\
 - \quad 9 \\
 \hline
 18 \\
 - \quad 9 \\
 \hline
 9 \\
 - \quad 9 \\
 \hline
 0
 \end{array}
 \end{array}
 \begin{array}{l}
 (1 \times 9) \\
 (1 \times 9) \\
 (1 \times 9)
 \end{array}$$

So 3 lots of 9 fit into 27; $27 \div 9 = 3$

We begin to use our tables to help us take larger chunks at a time:

$72 \div 6$

$$\begin{array}{r} 72 \\ - 60 \quad (10 \times 6) \\ \hline 12 \\ - 12 \quad (2 \times 6) \\ \hline 0 \end{array}$$

So, 10 + 2, or **12** lots of 6 fit into 72; $72 \div 6 = 12$

And then progress to larger numbers, using place value:

$172 \div 5$

$$\begin{array}{r} 175 \\ - 150 \quad (30 \times 5) \\ \hline 25 \\ - 25 \quad (5 \times 5) \\ \hline 0 \end{array}$$

So, 30 + 5, or **35** lots of 5 fit into 175; $175 \div 5 = 35$

There may sometimes be remainders:

$196 \div 6$

$$\begin{array}{r} 196 \\ - 180 \quad (30 \times 6) \\ \hline 16 \\ - 12 \quad (2 \times 6) \\ \hline 4 \end{array}$$

So, 30 + 2, or 32 lots of 6 fit into 196, with 4 left over;

$196 \div 6 = 32 \text{ r. } 4$

Moving on in Upper KS2

Minimum end of year expectations for Year 5

- Identify all multiples & factors, including finding all factor pairs.
- Use known tables to derive other number facts.
- Multiply: 4-digits by 1-digit then 2-digit
- Divide: 4-digits by 1-digit
- Multiply & divide: Whole numbers & decimals by 10, 100 & 1000
- Multiply proper fractions by whole numbers.

Multiplying by 10, 100 or 1,000 using Place Value

As the children's understanding of place value deepens, they can multiply by 10, then 100, 1,000 and further multiples of 10 by 'shuffling' the numbers, initially on a Hundreds, Tens and Units grid:

$$27 \times 10 =$$

H	T	U
	2	7
2	7	0

$$27 \times 10 = 270$$

It is important to emphasise that we **DO NOT** simply 'add a zero', but use the zero as a 'place holder'.

$$52 \times 100 = ?$$

Th	H	T	U
		5	2
5	2	0	0

$$52 \times 100 = 520$$

Eventually, $0.61 \times 100 = ?$

H	T	U	• t	h
			• 6	1
6	1	0	• 0	0

$$0.61 \times 100 = 610$$

Dividing by 10, 100 or 1,000 using Place Value

$$270 \div 10 =$$

H	T	U
2	7	0
	2	7

$$270 \div 10 = 27$$

Then: $5,200 \div 100 =$

Th	H	T	U
5	2	0	0
		5	2

$$5,200 \div 100 = 52$$

Eventually, $612 \div 100 =$

H	T	U	• t	h
6	1	2	• 0	0
		6	• 1	2

$$612 \div 100 = 6.12$$

Multiplying 4-digits by 1

We continue to use the grid method of multiplication to consolidate understanding of place value:

<i>x</i>	<i>1000</i>	<i>200</i>	<i>40</i>	<i>8</i>
<i>5</i>	<i>5000</i>	<i>1000</i>	<i>200</i>	<i>40</i>

And then use column addition to calculate the answer:

$$\begin{array}{r}
 5000 \\
 1000 \\
 200 \\
 + 40 \\
 \hline
 6240
 \end{array}$$

$$1,248 \times 5 = 6,240$$

Multiply up to 4-digit by 2-digit numbers

The grid method is used at first:

<i>x</i>	<i>100</i>	<i>40</i>	<i>8</i>
<i>10</i>	<i>1000</i>	<i>400</i>	<i>80</i>
<i>5</i>	<i>500</i>	<i>200</i>	<i>40</i>

$$\begin{array}{r}
 1480 \\
 + 740 \\
 \hline
 2220
 \end{array}$$

1 1

(carrying digits over below the line)

Then an expanded column method to keep track of all the parts of each number:

$$\begin{array}{r}
 148 \\
 \times 15 \\
 \hline
 40 \quad (5 \times 8) \\
 200 \quad (5 \times 40) \\
 500 \quad (5 \times 100) \\
 80 \quad (10 \times 8) \\
 400 \quad (10 \times 40) \\
 \underline{1000} \quad (10 \times 100) \\
 2220 \\
 11
 \end{array}$$

Children say the value of the digits not just the single digit: '5 x 40' not '5 x 4'.

Dividing 4-digit by 1-digit numbers

Children continue to secure the expanded 'chunking' method of division before being taught to use short division methods

We begin to use short division methods to divide whole numbers up to 4 digits by 1-digit numbers:

$$78 \div 3 = \begin{array}{r} 26 \\ 3 \overline{) 78} \end{array} \quad (\text{carrying the remainder across})$$

Minimum end of year expectations for Y6

- Compare & order numbers up to 10000000.
- Identify common factors, common multiples & prime numbers.
- Round any whole number to a required degree of accuracy.
- Multiply: 4-digit by 2-digit
- Divide: 4-digit by 2-digit
- Multiply simple pairs of proper fractions, writing the answer in the simplest form.
- Divide proper fractions by whole numbers.
- Calculate % of whole number

Children are taught to choose the most appropriate and efficient method for the calculations they need to do.

Multiplying 4-digit by 2-digit

When multiplying a 4-digit by a 1-digit number, children would continue to use the grid method or the short column method.

When multiplying a 3 or 4-digit by a 2-digit number they would use the grid method or an expanded column method, using their knowledge of place value and multiplication tables to make their calculation more efficient.

$$\begin{array}{r} 246 \\ \times 35 \\ \hline 7000 \\ 1400 \\ \underline{210} \\ 8610 \end{array} \quad \begin{array}{l} (200 \times 35) \\ (40 \times 35) \\ (6 \times 35) \end{array}$$

Dividing 4-digit by 2-digit numbers

Children will be taught to use the expanded method of division.

$972 \div 36$

$$\begin{array}{r} 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array} \quad \begin{array}{l} (20 \times 36) \\ (7 \times 36) \end{array}$$

$972 \div 36 = 27$

This method will also be used to teach division of decimals by a 1-digit number after approximating the answer:

$$87.5 \div 7 \quad (\text{approximately } 80 \div 8 = 10)$$

$$\begin{array}{r} 7 \overline{) 87.5} \\ - 70.0 \quad (10 \times 7) \\ \hline 17.5 \\ - 14.0 \quad (2 \times 7) \\ \hline 3.5 \\ - 3.5 \quad (0.5 \times 7) \\ \hline 0 \end{array}$$

$$87.5 \div 7 = 12.5$$

Multiply simple pairs of proper fractions, writing the answer in the simplest form:

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} \quad \frac{6}{12} = \frac{3}{6} = \frac{1}{2}$$

Divide proper fractions by whole numbers:

$$\frac{3}{4} \div 2 \quad \frac{3}{4} \div \frac{2}{1} \quad \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

Calculate % of whole numbers :

Children are taught to use doubling and halving, and their understanding of dividing by 10 or 100, to find % of numbers.

To find 35% of £180:

$$10 \% \text{ of } 180 = 18$$

$$\text{So } 5\% \text{ of } 180 = 9$$

$$18 \times 3 = 54$$

$$54 + 9 = 63$$

$$35\% \text{ of } 180 = \text{£ } 63$$

Bibliography:

The National Curriculum in England key stages 1 and 2 framework document' in September 2013.

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